## Chapter 6 Graphs of Trigonometric Functions Lab

For each of the angles below, calculate the values of $\sin x$ and $\cos x$ ( 2 decimal places) on the chart and graph the points on the graph below.

| $x$ | $0^{\circ}$ | $30^{\circ}$ | $45^{\circ}$ | $60^{\circ}$ | $90^{\circ}$ | $120^{\circ}$ | $135^{\circ}$ | $150^{\circ}$ | $180^{\circ}$ | $210^{\circ}$ | $225^{\circ}$ | $240^{\circ}$ | $270^{\circ}$ | $300^{\circ}$ | $315^{\circ}$ | $330^{\circ}$ | $360^{\circ}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y=\sin x$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $y=\cos x$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |



What you are seeing are the graphs of the sine and cosine functions. Since the function values repeat, you are looking at one cycle of the curves. We call these curves periodic because of their repetitive nature.

Let us look at these curves on the calculator. For right now, let's stay in degree mode.

|  |  |
| :---: | :---: |



Let's note the similarities and differences of the sine and cosine curves:

## Similarities

## Differences

Since the curves are similar in shape, we call them sinusoids.
Note that since these are periodic functions, we can look at as many periods as we wish:

4. Graphing Trig Functions

Also notice that we can graph both the sine and cosine function in radian mode:

|  |
| :---: |


the window was set up with Xmax as $2 \pi$ and Xscl as $\frac{\pi}{6}$
ZoomTrig (Degree) ZoomTrig (Radian)

While we will be exploring most of our graph at first in degree mode, please realize that graphs scaled correctly should be graphed in radian mode because you are comparing numbers on the $x$-scale to numbers on the $y$-scale. Graphing in Degree mode is comparing degrees on the $x$-scale to numbers on the $y$-scale.

When we graph a sinusoid within its primary period of $[0,2 \pi)$ or $\left[0^{\circ}, 360^{\circ}\right)$, there are 5 points that help us in sketching the curve. We call them the critical points of the sinusoid. By definition, the critical points are the points in which the curve either crosses it axis of symmetry, reached a high point (maximum) or low point (minimum).

Radians or Degrees

|  | 0 or $0^{\circ}$ | $\frac{\pi}{2}$ or $90^{\circ}$ | $\pi$ or $180^{\circ}$ | $\frac{3 \pi}{2}$ or $270^{\circ}$ | $2 \pi$ or $360^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y=\sin x$ | $(0,0)$ or $\left(0^{\circ}, 0\right)$ | $\left(\frac{\pi}{2}, 1\right)$ or $\left(90^{\circ}, 1\right)$ | $(\pi, 0)$ or $\left(180^{\circ}, 0\right)$ | $\left(\frac{3 \pi}{2},-1\right)$ or $\left(270^{\circ},-1\right)$ | $(2 \pi, 0)$ or $\left(360^{\circ}, 0\right)$ |
| $y=\cos x$ | $(0,1)$ or $\left(0^{\circ}, 1\right)$ | $\left(\frac{\pi}{2}, 0\right)$ or $\left(90^{\circ}, 0\right)$ | $(\pi,-1)$ or $\left(180^{\circ},-1\right)$ | $\left(\frac{3 \pi}{2}, 0\right)$ or $\left(270^{\circ}, 0\right)$ | $(2 \pi, 1)$ or $\left(360^{\circ}, 1\right)$ |




## Amplitude and Period

Now that we know the shape and behavior of the sine and cosines curves, we will now do some things to alter the behavior. Our goal is to predict the shape of the curve without resorting to actually graphing it.

On your calculators, graph the curves $y=\sin x, y=2 \sin x, y=4 \sin x, y=-.5 \sin x, y=-3 \sin x$ in degree mode on the window below.


Notice that by changing the coefficient of the function, we control its scaling factor - a vertical stretch or vertical shrink of the basic sine curve. We call this the amplitude of the curve - the height of the curve above its axis of symmetry.

The amplitude of $y=a \sin x$ or $y=a \cos x$ is the largest value of $y$ and is given by $|a|$.
The range of the curve is $[-a, a]$.
We define the shape of the curve using this chart: $\left\{\begin{array}{l}a>0 \ldots \text { curve normal } \\ a<0 \ldots \text { curve reversed }\end{array}\right.$
We define vertical change using this chart: $\left\{\begin{array}{l}\text { amplitude }>1 \ldots \text { vertically stretch } \\ \text { amplitude }<1 \ldots \text { vertically shrunk } \\ \text { amplitude }=1 \ldots \text { no vertical change }\end{array}\right.$

1) For each of the curves below, find the amplitude and range. Verify your results on a graphic calculator if available.

|  | Curve | Amplitude | Range | Shape (circle) | Vertical (circle) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| a. | $y=3 \cos x$ |  |  | Normal / Reverse | Stretch / Shrink |
| b. | $y=-2 \sin x$ |  |  | Normal / Reverse | Stretch / Shrink |
| c. | $y=\cos x$ |  |  | Normal / Reverse | Stretch / Shrink |
| d. | $y=7 \sin x$ |  |  | Normal / Reverse | Stretch / Shrink |
| e. | $y=\frac{2}{3} \sin x$ |  |  | Normal / Reverse | Stretch / Shrink |
|  |  |  |  |  |  |
| f. | $y=-\pi \cos x$ |  |  | Normal / Reverse | Stretch / Shrink |

On your calculators, graph the curves $y=\sin x, y=\sin 2 x, y=\sin 4 x, y=\sin .5 x$ in degree mode on the window on the next page.


Notice that by changing the coefficient of the $x$-variable, we control the number of cycles of the curve we produce. In $y=\sin x$, when the coefficient of $x$ is 1 , we get one cycle within $360^{\circ}$. In $y=\sin 2 x$, we get two cycles within $360^{\circ}$. Thus we also change the period of the curve, the number of degrees (or radians) the curve takes to complete one cycle.

The period of $y=\sin b x$ or $y=\cos b x$ is the number of degrees (or radians) the curve takes to complete one cycle. It is found using this formula: Period $=\left\{\begin{array}{l}\frac{360^{\circ}}{b} \text { for degrees } \\ \frac{2 \pi}{b} \text { for radians }\end{array}\right.$

We define the horizontal stretch in words: $\left\{\begin{array}{l}\text { period }<360^{\circ}(\text { or } 2 \pi) \ldots \text { compressed } \\ \text { period }>360^{\circ}(\text { or } 2 \pi) \ldots \text { elongated } \\ \text { period }=360^{\circ}(\text { or } 2 \pi) \ldots \text { none }\end{array}\right.$
To find the 5 critical points, we use $0, \frac{\text { period }}{2}$, period, $\frac{3 \text { period }}{2}$, period whether we are in degrees or radians.
2) For each of the curves below, find the period and horizontal stretch. Verify your results on a graphic calculator if available.

|  | Curve | Period | Horizontal Stretch（circle） |
| :--- | :--- | :--- | :--- |
| a． | $y=\cos 2 x$ |  | Compressed／elongated |
| b． | $y=\sin 3 x$ |  | Compressed／elongated |
| c． | $y=\cos 4 x$ |  | Compressed／elongated |
| d． | $y=\sin 0.5 x$ |  | Compressed／elongated |
| e． | $y=\cos \frac{2}{3} x$ |  | Compressed／elongated |
| f． | $y=\sin \frac{3}{2} x$ |  | Compressed／elongated |
| g． | $y=\cos 10 x$ |  | Compressed／elongated |

3）Now lets put both amplitude and period together．For each of the curves below，find all the pertinent information．Verify your results on a graphic calculator if available．

|  | Curve | Amplitude | Period | Shape （circle） | Vertical <br> Stretch（circle） | Horizontal Stretch（circle） |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| a． | $y=2 \sin x$ |  |  | Normal Reversed | Stretched Shrunk | Compressed Elongated |
| b． | $y=\sin 2 x$ |  |  | Normal Reversed | Stretched Shrunk | Compressed Elongated |
| c． | $y=2 \sin 2 x$ |  |  | Normal Reversed | Stretched Shrunk | Compressed Elongated |
| d． | $y=4 \cos 3 x$ |  |  | Normal Reversed | Stretched Shrunk | Compressed Elongated |
| e． | $y=-3 \cos 6 x$ |  |  | Normal Reversed | Stretched Shrunk | Compressed Elongated |
| f． | $y=\frac{1}{2} \sin \frac{1}{2} x$ |  |  | Normal Reversed | Stretched Shrunk | Compressed Elongated |
| g． | $y=\sin \frac{4}{3} x$ |  |  | Normal Reversed | Stretched Shrunk | Compressed Elongated |
| h． | $y=-\frac{3}{4} \sin \frac{3}{4} x$ |  |  | Normal Reversed | Stretched Shrunk | Compressed Elongated |

## Translations of Sinusoids

On your calculators，graph $y=\sin x, y=2+\sin x, y=-1+\sin x$ ．


WIFLIDTO
人口ir＝宣
$x \Gamma \dot{\theta}=6$
人
Y币if＝
以币白 $\times 2$
$y=c 1=.1$
xres＝1

Note by changing the constant that is added or subtracted to the basic sin or cosine curve, we affect how the graph of the sinusoid is shifted up or down. This is called vertical translation or vertical shift.

> A curve in the form of $y=d \pm a \sin b x$ or $y=d \pm a \cos b x$ will shift the curves $y= \pm a \sin b x$ or $y= \pm a \cos b x$ up or down based on the value of $d$. This is called the vertical translation of the curve. $\left\{\begin{array}{l}\text { If } d>0 \ldots \text { vertical translation is up } d \text { units } \\ \text { If } d<0 \ldots \text { vertical translation is down } d \text { units } \\ \text { If } d=0 \ldots \text {..there is no vertical translation }\end{array}\right.$

Finally, on your calculators, graph the functions $y=\sin x, y=\sin \left(x-30^{\circ}\right), y=\sin \left(x+45^{\circ}\right)$


Note by changing the constant that is added or subtracted within the parentheses, we shift the sinusoid left or right. This is called horizontal translation or phase shift.

A curve in the form of $y=\sin (x-c)$ or $y=\cos (x-c)$ will shift the sinusoid right or left based on the value of $c$. The value of $c$ is the phase shift (or horizontal translation).
$\int$ If $c>0$, the curve shifts right $\ldots$ so $y=\sin \left(x-10^{\circ}\right)$ shifts the curve right
If $c<0$, the curve shifts left $\ldots$ so $y=\sin \left(x+10^{\circ}\right)$ shifts the curve left
When there is a phase shift, the critical points change as well. We call these the true critical points. We find the critical points and add or subtract the phase shift to get the true critical points.

$$
\begin{aligned}
& \text { Critical Points }=\quad 0, \frac{\text { period }}{4}, \frac{\text { period }}{2}, \frac{3 \text { period }}{4}, \text { period } \\
& \text { True Critical Points }=c, \frac{\text { period }}{4}+c, \frac{\text { period }}{2}+c, \frac{3 \text { period }}{4}+c, \text { period }+c
\end{aligned}
$$

4) For each curve, find the vertical translation and phase shift. Verify your results on a graphic calculator if available.

|  | Curve | Vertical <br> Translation | Phase shift / <br> direction |
| :--- | :--- | :--- | :--- |
| a. | $y=4+\sin x$ |  |  |
| b. | $y=\cos \left(x-60^{\circ}\right)$ |  |  |
| c. | $y=5+\sin \left(x+20^{\circ}\right)$ |  |  |
| d. | $y=-2-\cos \left(x+\frac{\pi}{6}\right)$ |  |  |
| e. | $y=\sin \left(x-\frac{\pi}{4}\right)-1$ |  |  |

Let's put it all together. Given the curves $y=d \pm a \sin b(x-c)$ or $y=d \pm a \cos b(x-c)$
The amplitude $=|a|$
We define the shape of the curve using this chart: $\left\{\begin{array}{l}a>0 \ldots \text { curve normal } \\ a<0 \ldots \text { curve reversed }\end{array}\right.$
We define vertical change using this chart: $\left\{\begin{array}{l}\text { amplitude }>1 \ldots \text { vertically stretch } \\ \text { amplitude }<1 \ldots \text { vertically shrunk }\end{array}\right.$
The period $=\frac{360^{\circ}}{b}($ degrees $)$ or $\frac{2 \pi}{b}$ (radians)
We define the horizontal stretch in words: $\left\{\begin{array}{l}\text { period }<360^{\circ}(\text { or } 2 \pi) \ldots \text { compressed } \\ \text { period }>360^{\circ}(\text { or } 2 \pi) \ldots \text { elongated }\end{array}\right.$
The phase shift (or horizontal translation) $=c$.
If $c>0$, the curve shifts right, if $c<0$, the curve shifts left
Critical Points $=0, \frac{\text { period }}{4}, \quad \frac{\text { period }}{2}, \quad \frac{3 \text { period }}{4}$, period
True Critical Points $=c, \frac{\text { period }}{4}+c, \frac{\text { period }}{2}+c, \frac{3 \text { period }}{4}+c$, period $+c$
The Vertical Translation $=d$
$\left\{\begin{array}{l}\text { If } d>0 \ldots \text { vertical translation is up } d \text { units } \\ \text { If } d<0 \ldots \text { vertical translation is down } d \text { units }\end{array}\right.$
The range of the function is $[d-a, d+a]$
5) For each curve, find all pertinent information and verify your results on a graphic calculator if available.

|  | Curve: | Amplitude | Pd. | Phase Shift \& Direction | Vert Tran | Range | Shape (circle) | Vertical Stretch (circle) | Horizontal Stretch (circle) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| a. | $y=3-\cos x$ |  |  |  |  |  | Normal Reversed | Stretched Shrunk | Compressed Elongated |
| b. | $y=1+2 \cos 2 x$ |  |  |  |  |  | Normal Reversed | Stretched <br> Shrunk | Compressed <br> Elongated |
| c. | $y=-2-3 \sin 4 x$ |  |  |  |  |  | Normal Reversed | Stretched Shrunk | Compressed Elongated |
| d. | $y=5 \sin \left(x-30^{\circ}\right)$ |  |  |  |  |  | Normal Reversed | Stretched <br> Shrunk | Compressed Elongated |
| e. | $y=-1-\cos \left(x+15^{\circ}\right)$ |  |  |  |  |  | Normal Reversed | Stretched Shrunk | Compressed Elongated |
| f. | $y=2+\frac{1}{2} \sin 2\left(x-5^{\circ}\right)$ |  |  |  |  |  | Normal Reversed | Stretched Shrunk | Compressed <br> Elongated |
| g. | $y=\frac{1}{2}-4 \cos \left(3 x+15^{\circ}\right)$ |  |  |  |  |  | Normal Reversed | Stretched Shrunk | Compressed <br> Elongated |
| h. | $y=-\frac{3}{4} \sin \left(\frac{1}{2} x-40^{\circ}\right)$ |  |  |  |  |  | Normal Reversed | Stretched <br> Shrunk | Compressed Elongated |
| i. | $y=5+3 \sin \left(2 x-\frac{\pi}{2}\right)$ |  |  |  |  |  | Normal Reversed | Stretched <br> Shrunk | Compressed Elongated |

## Determining an equation from the graph of a sinusoid



Suppose you were given this graph. Could you determine its equation? Here are some questions you need to answer.

Is it a sine curve or cosine curve? $\qquad$
Where is its axis of symmetry? $\qquad$
What is its amplitude? $\qquad$ What is its period? $\qquad$

Here is the generalization to determine the equation from its graph: Do the problem above on the right. The equation will be in the form $y=d \pm a \sin b(x-c)$ or $y=d \pm a \cos b(x-c)$

1) Decide whether it is a sine or cosine curve. If it "starts" at a high point or low point, it is a cosine curve. If it "starts" in the middle, it is a sine curve. You also must determine if the curve is reversed. If so $a<0$.
2) Draw the axis of symmetry. That is the value of $d$.
3) Find the height of the curve above the axis of symmetry. That is $a$.
4) Find the period by inspection. period $=\frac{360^{\circ}}{b}$ then $b=\frac{360^{\circ}}{\text { period }}$
5) Is there a shift? If shifted right, $c>0$. If shifted left, $c<0$.
6) Put it all together.
